Algebra II 6.3 Logarithmic Functions

Obj: Evaluate and simplify logarithms

Example 1. Solve $2^x = 8$ **what would happen if you had** $2^x = 10$

$2 \cdot 2 \cdot 2 = 8$

$2^3 = 8$

$x = 3$

hmm...
would be between
3 + 4
$2^3 = 8$ $2^4 = 16$
guess+check?
no.....

The inverse of an exponential function is called a logarithm.

take note

Key Concept Logarithm

A **logarithm** base b of a positive number x satisfies the following definition.

$\log_b x = y$ if and only if $b^y = x$

$b^y = x$

circle of life

For a logarithm, $b > 0$ $x > 0$ $b \neq 1$ b is the base.

$b^y = x$ $\log_b x = y \rightarrow$ is the exponent

Example 2. Rewriting.
What is the logarithmic form of each equation?

a. $81 = 3^4$ $\log_3 81 = 4$

b. $\frac{8}{27} = \left(\frac{2}{3}\right)^3$ $\log_{2/3} \frac{8}{27} = 3$

c. $1 = 3^0$ $\log_3 1 = 0$

Rewriting a log to an exponential function.

a. $\log_{10} 1000 = 3$ $10^3 = 1000$

b. $\log_{\frac{1}{3}} 9 = -2$ $\frac{1}{3}^{-2} = 9$

c. $\log_x y = z$ $x^z = y$

Example 3: Evaluating a Logarithm. What is the value of each logarithm?

a. $\log_5 125 = x$ $5^x = 125$ $x = 3$

b. $\log_{\frac{1}{4}} 16$ $\frac{1}{4}^x = 16$ $x = -2$

c. $\log_3 0 = x$ $3^x = 0$ never !!

d. $\log_2 \frac{1}{8} = x$ $2^x = \frac{1}{8}$ $x = -3$

e. $\log_{64} \frac{1}{32} = x$ $64^x = \frac{1}{32}$ $(2^6)^x = 2^{-5}$ $6x = -5$ $x = -5/6$

f. $\log_{27} 3 \rightarrow 27^x = 3$ $x = 1/3$

Properties of logs.

$\log_b b^x = x$ example: $\log_8 8^5 = 5$

same

$b^{\log_b x} = x$ example: $3^{\log_3 8} = 8$

same

$\log_b 1 = 0$ example: $\log_6 1 = 0$ all log 1 = 0

Simplify: $\log_7 7^4 + 5^{\log_5 3}$

$4 + 3 = 7$

There are two log functions on your calculator.

Special Logarithms

Common Logarithm	Natural Logarithm
➢ Logarithm with base 10	➢ Logarithm with base *e*
➢ Denoted by: $\log_{10} x$	➢ Denoted by: $\log_e x$
➢ Simplified Notation: $\log x$	➢ Simplified Notation: $\ln x$

Special Note: your calculator has keys for evaluating the common and natural logarithm.

Example 4. Use your calculator to evaluate.

Log900	log1000	ln 5	ln e	log(-7)
2.9542	3	1.609	1	DNE

can't log a negative # or 0.

Example 5. Solve equations with logs. (usually involves rewriting to the other form). **Isolate the log or base first!**

a. $10^{x-1} = 25$

$\log 10^{x-1} = \log 25$

$x-1 = \log 25$

$x = \log 25 + 1$

$x \approx 2.398$

b. $\ln(2x+3) = 4$

$e \qquad e$

Use e as base of both

$2x+3 = e^4$

$2x = e^4 - 3$

$x = \dfrac{e^4-3}{2} \approx 25.799$

You try.

a. $\dfrac{2e^{x+2}}{2} = \dfrac{16}{2}$

$e^{x+2} = 8$

$\ln e^{x+2} = \ln 8$

$x+2 = \ln 8$

$x = \ln 8 - 2 \approx$

b. $\log(3x-2) = 2$

$10 \qquad 10$

raise 10 as base of both

$3x-2 = 10^2$

$3x-2 = 100$

$3x = 102$

$x = 34$

Extraneous solutions for logs…. → only if start w/ log

if your value when starting with a log results in taking a negative or zero log

Example 6. The seismic energy, ***x***, in joules can be estimated based on the magnitude, ***m***, of an earthquake by the formula $x = 10^{1.5m+12}$. What is the magnitude of an earthquake with a seismic energy of $4.2 \bullet 10^{20}$ joules?

$x = 10^{1.5m+12}$

$4.2 \cdot 10^{20} = 10^{1.5m+12}$

$\log \qquad \log$

$\log 4.2 \cdot 10^{20} = 1.5m+12$

$20.6 \approx 1.5m+12$

$m \approx 5.75$ magnitude